Electric Probes in Stationary and Flowing Plasmas: Part 1. Collisionless and Transitional Probes

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Nomenclature

$\hat{a}$ = defined in Eq. (17), part 2
A_p = probe area
c = particle mean random thermal speed
C = mass fraction
D = diffusion coefficient
$\mathscr{D}, \mathscr{D}_e$ = Damkohler numbers $k_r L^2 N_{eo}{}^2/D_{io}$, and $N_{eo} 2m_e \langle C_e \rangle / L m_n \lambda_{en}$, respectively
e = electronic charge
E = electric field; also total energy, Eq. (4), part 1
f = Blasius function
H = enthalpy
I = actual current collected by probe
j = normalized current density
J = current per unit area
$\tilde{j}_s$ = defined in Eq. (15)
k = Boltzmann constant
K_{he} = electron thermal conductivity
Kn = Knudsen number, λ/R
l = probe length
m = particle mass
M = Mach number
$\mathscr{M}$ = surface ion concentration, Eq. (15), part 2
N = number density
P = pressure
Pr = Prandtl number, $\mu c_p/k$
Q = collision cross section
r = radial distance
R = probe radius
Re = Reynolds number, $\rho UL/\mu$
Rs = defined in Eq. (12), part 1
Sc = Schmidt number, $\mu/\rho D$
Sc_i = ion Schmidt number
t = time
T = temperature
$U_i(\ )$ = potential energy, Eq. (3), part 1
u, U = flow velocity
V_r, V_θ = radial and azimuthal velocity
$\dot{w}$ = source term, Eqs. (1) and (2), part 2
$\dot{w}_{he}$ = energy source term, Eq. (3), part 2
x = axial coordinate
y = coordinate normal to probe surface
Z = charge number

Greek Letters

α = exponent defined in Eq. (16), part 1, also ratio λ_D/L
β = E/kT_i, also Eq. (16), part 1; ratio of diffusion coefficients, D_i/D_e, in part 2
κ = a parameter $[= (\lambda_D{}^2/L^2) Re\, Sc_i]^{-1} \simeq \delta^2/\lambda_D{}^2 \propto \hat{a}$ in part 2
δ = boundary-layer thickness
ε_e = electron-heavy gas collision term
ε = T_i/T_e
η = similarity variable
$\lambda_{\alpha\beta}$ = mean free path for collisions between species α and β
λ_D = Debye shielding length, $(\sigma_o kT_e/e^2 N_\infty)^{1/2}$
λ_s = sheath thickness
ν_i = ion-neutral collision frequency[20]
ξ = r/λ_D

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ξ_p = R/λ_D, Debye ratio
$\mu_{i,e}$ = mobility (ion and electron)
μ = viscosity
ρ = gas density
σ_o = permittivity of free space
τ = temperature ratio T_e/T_i in part 2, also $(x/l)\tau_l = \omega_{pi} t$ in part 1
τ_l = parameter defined in Eq. (20)
ϕ = electric potential
χ, χ^* = normalized potentials $Ze\phi/kT_i$ and $Ze\phi/kT_e$, respectively
χ_p = normalized probe potential
ψ = nondimensional potential, $-\phi/\phi_p$
ω_{pi} = ion plasma frequency, $(N_\infty e^2/\sigma_o m_i)^{1/2}$
Ω = angular momentum $m_i r V_\theta$

Subscripts

e, i, n = electron, ion, neutral
en, in = electron-neutral, ion-neutral
o = conditions at reference point
p = probe
s = sheath edge conditions
sat = saturation, point a in Fig. 2, pt. 1
w = wall
δ = boundary-layer edge conditions
∞ = freestream conditions

Symbols without subscript e, i, or n refer to the mixture.

Introduction

THE electric probe has long been used as a fundamental diagnostic tool for measuring the local properties of a plasma. The pioneering work in the use of electric probes was done by Langmuir in 1924; consequently, these probes are often called Langmuir probes. Electric probes are relatively simple devices, but the theory underlying the probe response is, unfortunately, complicated. Basically, the electric probe consists of one or more small metallic electrodes inserted into a plasma. Two probe configurations are commonly employed. In the "single probe" configuration, a single electrode is inserted into the plasma and attached to a power supply which can be biased at various potentials positive or negative relative to the plasma, and the current collected by the probe as a function of its potential is measured. The return electrode which completes the circuit is a "ground" for the plasma, and is typically a conducting portion of the wall of the vessel confining the plasma. In this case the probe potential is measured from the ground potential. In the typical "double probe" configuration, two electrodes commonly of equal area are in contact with the plasma, and the current passing through the plasma between the two electrodes is measured as a function of the voltage applied between them. Unlike the single probe, in the case of the double probe there is no net charge drain from the plasma, since the two electrodes and power supply of the double probe system form an isolated closed circuit. Also, unlike the single probe, both electrodes of a double probe system are always (in the case of an ion-electron plasma) negative with respect to the plasma. There are certain advantages associated with the use of each of the probe systems, as will be discussed subsequently.

The reason electric probe theory is complicated stems from the fact that probes are boundaries to plasmas, and near boundaries the equations that govern the plasma behavior change. Charge neutrality does not hold near boundaries and a thin layer exists where electron and ion number densities differ; the layer, often called a Debye sheath, can sustain large electric fields. In the absence of magnetic fields, the response of a probe in a flowing plasma depends on a number of parameters which arise naturally from the governing equations. These parameters may be combined into groups as indicated in Table 1.

The two parameters in Group I determine the various domains where an electric probe can operate. The domains are given schematically in Fig. 1 where the relative magnitudes of R, λ, and λ_D determine six regimes of probe operation under two major domains,* viz:

A) $Kn \gg 1$: Thin-wire Langmuir probe—1) $\lambda \gg R \gg \lambda_D$—conventional thin sheath; 2) $\lambda \gg \lambda_D \gg R$—orbital limit—thick sheath; 3) $\lambda_D \gg \lambda \gg R$—collisional thick sheath.

B) $Kn \ll 1$: Continuum Electric Probe—1) $R \gg \lambda_D \gg \lambda$—collisional thin sheath; 2) $\lambda_D \geqq R \gg \lambda$—collisional thick sheath; 3) $R \gg \lambda \geqq \lambda_D$—collisionless thin sheath (dense case).

Of course, one can identify in the preceding regimes two transitional regions, $Kn = 0(1)$ and $\lambda_D \simeq \lambda$ and the double-transitional case where $R \simeq \lambda_D \simeq \lambda$.

In Group II, the probe potential χ_p determines the positive and negative charge collection modes; T_i/T_e and the Damkohler number, $\mathscr{D}_e$, represent the level of thermal equilibrium in the plasma between electron and heavy particles; and the Damkohler number, $\mathscr{D}$, gives the degree of chemical nonequilibrium.

The Group III parameters have, by and large, a secondary influence on probe characteristics (with certain exceptions) and often depend on the particular probe geometry under consideration.

The literature on probes is vast. Chen[1] in 1965 was the first to summarize the available theoretical results in a systematic manner and supplement them with practical information on experimental techniques. Since that time there has been an accelerated activity in the study of electric probes for arc jet environments, on re-entry vehicles and on satellites for measuring charged particle densities and electron temperatures.

In this Survey, we will attempt to review the state-of-the-art of electric probes. The survey will be in two parts. Part 1 will be devoted to collisionless and transitional studies, and part 2 will cover the continuum regime for probe operation. Whenever possible, results will be given in forms suitable for interpreting probe data both in ground facilities and flight conditions.

Figure 2 depicts the general shape of a current-voltage (CV) characteristic curve of a single probe in a typical plasma environment. This plot can be obtained in a continuous fashion in a steady-state plasma or even in a transient plasma by the use of a fast-sweeping voltage source.

The qualitative behavior of the CV characteristic depicted in Fig. 2 can be explained as follows. For large negative values of the probe potential $\phi_p < \phi_{p,a}$, essentially all electrons in the vicinity of the probe are repelled from it. The electron current

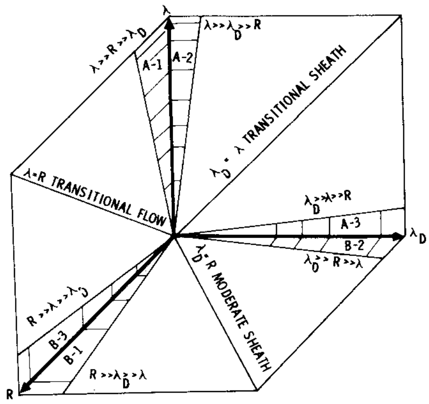

Fig. 1 Three-dimensional diagram illustrating various probe operation regimes.

Table 1 Governing parameters

Group I	λ_D/R; λ/R
Group II	χ_p; T_i/T_e; $\mathscr{D}$, $\mathscr{D}_e$
Group III	Continuum probes: Sc_i; Re; M; T/T_w
	Collisionless-transition probes: l/R; U/c

* For the moment, we use λ to denote a representative mean free path, without specifically defining the collision parameters.

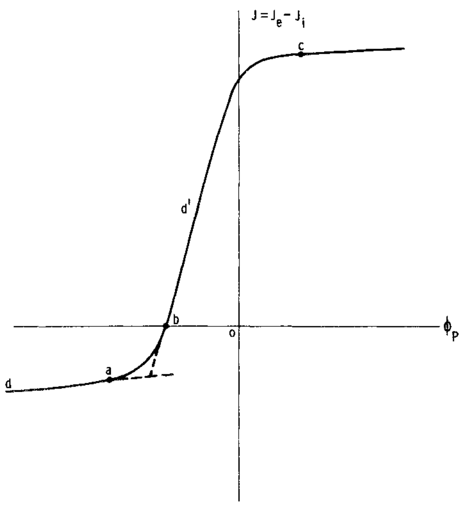

Fig. 2 Typical CV characteristic curve for single probe operation.

to the probe is negligible and the electric current to the probe consists of the ion current j_i, which is of the order of the natural ion diffusion current. Although j_i will, in general, continue to increase for $\phi_p \leqq \phi_{p,a}$, this branch of the characteristic is called the "ion saturation" current, j_{is}. The nature of this "saturation" depends on the governing parameters previously discussed, in particular, on whether the plasma is collisional or collisionless, relative to the probe. In many cases, j_{is} can be conveniently related to the electron number density N_e provided (T_e/T_i) is known.

When ϕ_p is made less negative relative to the plasma than its value at point a, the most energetic electrons in the plasma are able to overcome the probe's retarding electric field and hence reach the probe, giving rise to an electron current contribution to the measured probe current which decreases the net current observed. As the probe potential is made still less negative, a condition is reached at point b where the electron current collected exactly balances the ion current, and the net current to the probe is zero. This point is called the floating potential. At the floating potential, the probe is still negative with respect to the plasma, and collects essentially the saturation-ion current as well as a cancelling electron current.

In the region between $\phi_{p,b}$ and $\phi_{p,c}$, the probe potential is still negative, relative to the plasma potential $\phi_{p,c}$, but the probe attracts increasingly more electrons so that the net current to the probe is an electron current. If the electron distribution were Maxwellian, the shape of the curve here, after the ion contribution is subtracted, would be exponential and from the shape of this curve T_e could be determined.

Near $\phi_p = \phi_{p,c}$, the plasma potential, the electric field of the probe approaches zero and the electron current increases toward its natural diffusive value which is of order $(m_i T_e/m_e T_i)^{1/2}$ times the ion-saturation current. For $\phi > \phi_{p,c}$, the probe is at a potential which is positive relative to the plasma potential $\phi_{p,c}$ and j increases slowly as ions are repelled and electrons accelerated to the probe, giving rise to the "electron saturation" phenomena.

The determination of the location of $\phi_{p,c}$ on a CV characteristic where the probe is at the same potential as the plasma presents one of the more difficult problems of probe diagnostics. Very often there is no sharp discontinuity between the electron-saturation region j_{es} and the region between $\phi_{p,a}$ and $\phi_{p,c}$ (often called electron-retarding region). When plotted on a semi-logarithmic scale $\phi_{p,c}$ can be located approximately by extrapolating the linear part of the characteristic to meet the best "straight" line obtained from the region $\phi_p > \phi_{p,c}$. (For further discussion on this point, see Ref. 2.) A further difficulty in the determination of the plasma potential is due to the fact that often the grounding of the plasma is not ideal, in the sense that the plasma potential does not exactly coincide with ground potential, and a small positive probe potential relative to ground is required to obtain the natural electron diffusion current. These preliminary remarks are given in a cautionary vein to indicate that probe measurements which rely crucially on an exact determination of the plasma potential are likely to be less reliable than those for which a modest uncertainty in its determination can be tolerated.

When there is thermal nonequilibrium in the plasma with $T_e > T_i$, the prediction of electron saturation current is much more difficult than the ion-saturation current. This is because T_e increases with positive ϕ_p as the field does work on the electrons. For this reason the portion of the CV characteristics where $\phi_p < \phi_{p,c}$ is generally the most useful for diagnostics in continuum plasmas.

The behavior of a double probe can be determined from a knowledge of the single probe behavior of its components, and hence need not be treated separately. For example, suppose Fig. 2 represents the response of one of the two equal-area electrodes of a double probe when it is used as a single probe. Then in double probe operation if one electrode were operating at point d on the characteristic, the other electrode would have to operate at point d', such that $j_d = -j_{d'}$. In what follows, we will be concerned with single probe response, unless otherwise noted.

Part 1. Collisionless and Transitional Electric Probes

In this part of the Survey we will consider the response of electric probes operating in the collisionless and transitional regimes, starting with the quiescent (nonflowing) plasmas and going on to plasmas that possess a directed flow velocity. In terms of domains previously discussed, we are concerned here with the three cases delineated by A.1, A.2, and A.3. Our main attention will be directed toward simple geometries, namely spherical and cylindrical probes, although mention will be made of other geometries.

It will be useful at the outset to recall some of the early theories of probe response, particularly the pioneering work of Langmuir and his collaborators. Excellent and quite complete accounts of this material have been given by Chen[1] and by Swift and Schwar,[2] so we shall not attempt to be exhaustive here, but rather shall confine ourselves to the mention of results which are most important for our subsequent discussions of the more recent theories. Although the early theories have for the most part been superseded by the more recent ones, they in some instances represent valid limiting cases of the more general results, and therefore are still of interest.

For the sake of definiteness, we shall generally assume that the ions are singly charged, and are the species attracted to the probe, although the results we will present are equally applicable to electron-attracting probes, and can be scaled so as to apply to multiply charged ions. We define the normalized current densities j_i and j_e collected by the probe according to the relationship

$$I_{i,e} = A_p N_\infty Z_{i,e} e(kT_{i,e}/2\pi m_{i,e})^{1/2} j_{i,e} \tag{1}$$

We note that the factors $J_{R(i,e)} = N_\infty Z_{i,e} e(kT_{i,e}/2\pi m_{i,e})^{1/2}$ are the random thermal ion and electron currents collected by a probe at the plasma potential in a Maxwellian plasma so that the normalization consists of referring the probe current densities $I_{i,e}/A_p$ to these random currents

$$j_{i,e} = (I_{i,e}/A_p)/J_{R(i,e)} \tag{2}$$

However, it is often more convenient to normalize the probe ion current with respect to the random ion current evaluated at the electron temperature, in which case we denote the normalized ion current j_i^*, and observe that $j_i^* = j_i(\varepsilon)^{1/2}$, where $\varepsilon = T_i/T_e$. The reason this normalization is particularly useful in that it relates back to the so-called "Bohm condition"[3] which establishes the result that, for an ion-attracting probe, electrical fields in the quasi-neutral region exterior to the sheath accelerate ions such that they enter the sheath with approximately the velocity $(kT_e/m_i)^{1/2}$. Hence the random ion current evaluated at the electron temperature can be expected to provide a better reference for the actual ion current than a random current evaluated at the ion temperature.

Orbital Motion Limit

One of the most important early results concerns current collection by spherical and cylindrical probes in the orbital-motion-limit (OML), a terminology introduced by Langmuir and Mott-Smith,[4] who gave the results for this limit for charged particles having both monoenergetic and Maxwellian velocity distributions. The OML current is that current collected by a probe when none of the particles which come from the undisturbed plasma at infinity with the capability of reaching the probe on the basis of energy and angular momentum considerations are excluded from doing so by intervening barriers of effective potential. For the attracted particles, the OML corresponds to the limit $R/\lambda_D \to 0$, that is, the infinitely thick sheath limit.

As is well known, the two-body, central-force problem can be reduced to an equivalent, one-dimensional problem in which only the radial velocity v_r of the particle is considered if the local potential $\phi(r)$ is replaced by an effective potential, which for the case of ions is

$$U_i(r,\Omega) = Ze\phi(r) + (\Omega^2/2m_i r^2) \tag{3}$$

where Ze is the charge on the ion, and Ω is the angular momentum of the ion $(m_i r V_\theta)$ particle and is an invariant of the motion. Since the total energy of a particle is

$$E = \tfrac{1}{2}m_i(v_r^2 + v_\theta^2) + Ze\phi(r) = \tfrac{1}{2}m_i v_r^2 + U_i(r,\Omega) \tag{4}$$

and this total energy is an invariant of its motion, it follows that a particle starting in from infinity with particular values of E and Ω will reach a particular radius r only if $E - U_i(r) \geqq 0$, since otherwise v_r would be imaginary. Now the special feature of the orbital-motion-limit is that if at a particular radius r_1 the condition $E - U_i(r_1) \geqq 0$ is satisfied for a particle, then it is also satisfied for that particle for all $r > r_1$. In other words, there are no locations $r > r_1$ for which the effective potential $U_i(r)$ has a local maximum where $E - U_i(r) < 0$, which would act as a potential barrier to reflect particles back from their inward motion, even though the particles had sufficient energy to satisfy the condition $E - U_i(r_1) \geqq 0$ at an inner radius, which indeed could be the radius of the probe. It is, of course, not obvious under what conditions $U_i(r)$ will not have a local maximum, since it is a combination of a positive term $\Omega^2/2m_i r^2$ and a term $Ze\phi(r)$ which, in the case of an attracting probe, will be negative, but whose magnitude must be determined by a self-consistent solution of Poisson's equation. This, in fact, is the crux of the difficulty involved in the exact solution of the general collisionless probe problem. However, if it is assumed a priori that potential barriers do not exist, then the calculation of the current collected by a probe involves only energy and angular momenta considerations, and does not require a simultaneous solution of Poisson's equations. It can in fact be shown that the condition required for absence of a local maximum in $E - U_i$ is that $\phi(r)$ decrease less rapidly than r^{-2} with increasing r.

Considering ions to be the attracted species $(\chi_p < 0)$, we have the following infinite sheath results in the orbital-motion-limited current, for a Maxwellian† distribution at infinity:

spherical probe

$$j_i = 1 - \chi_p \tag{5a}$$

$$j_e = e^{\chi_p^*} \tag{5b}$$

cylindrical probe

$$j_i = 2/(\pi)^{1/2}\{(-\chi_p)^{1/2} + [(\pi)^{1/2}/2]\,e^{-\chi_p}[1 - \mathrm{erf}(-\chi_p)^{1/2}]\} \tag{6a}$$

$$j_e = e^{\chi_p^*} \tag{6b}$$

† If attracted species have monoenergetic velocity distribution, Eqs. (5a) and (6a) simplify to $j_i = 1 - (\pi/4)\chi_p$, and $[1 - (4/\pi)\chi_p]^{1/2}$, respectively (with $|\chi_p| \geqq 4/\pi$ and $\pi/4$, respectively).

All these expressions apply as well for the case where electrons are the attracted species, if interchange is made between subscripts i and e and the factors $-\chi_p$ and χ_p^*.

Recently, Laframboise and Parker[5] have re-examined the orbital-motion-limited regime, and rederived the results previously given on the basis of energy considerations alone. In so doing, they are able to establish that the results apply not only to circular cylinders and spheres, but to any convex cylindrical shape and to certain classes of sufficiently convex, three-dimensional shapes. In this connection it is of interest to note, as can be ascertained from Eqs. (5a) and (6a), that in the orbital-motion-limit the slope of the current-voltage characteristic of a spherical probe is proportional to $N_\infty/(T)^{1/2}$ (where T is the temperature of the attracted species), whereas for a cylindrical probe at large probe potentials the slope is proportional to N_∞. Thus, if both a spherical and a cylindrical probe are operated simultaneously, it should in principle be possible to determine N_∞, T_i, and T_e for the plasma, T_e being found either in the classical fashion from the slope of the retarding field portion of the characteristic or by choosing electrons as the attracted species. The difficulty in achieving this in practice is due to the aforementioned fact that the OML for a spherical probe is reached asymptotically only as $\xi_p \to 0$, which implies the requirement of a very tenuous plasma for a probe of practical size.[5]

However, even if the conditions in a plasma do not correspond to the OML regime for a given probe, the OML results are still of importance in that they provide an upper bound for the current collected by a probe under collisionless conditions. The reason for this is that potential barriers, which occur at finite values of R/λ_D, can only reduce the number of charged particles which are able to reach the probe.

The Cold Ion Approximation

Since in many plasmas $T_i/T_e \ll 1$, it is natural to examine the nature of probe response in the limit $T_i/T_e = 0$. This was done by Allen, Boyd, and Reynolds[6] for the case of a spherical probe in a collisionless plasma. They started with the equations of Bohm, Burhop and Massey,[3] which are the same as the equations later used by Bernstein and Rabinowitz,[7] and which apply for monoenergetic ions. In the case of an ion-attracting spherical probe, Allen et al. show that it is correct to assume that in the limit $T_i/T_e = 0$ the ions move in a radially inward direction, and their velocity is due solely to the energy they acquire in the potential field of the probe, i.e., $v_r = (-2e\phi/m_i)^{1/2} \equiv v_s(-\chi^*)^{1/2}$, where $v_s = (2kT_e/m_i)^{1/2}$. Since the current to the probe is $I_i = 4\pi r^2 N_i v_r$, the ion density at any radius r is by continuity

$$N_i = I_i/[4\pi r^2 v_s(-\chi^*)^{1/2}] \tag{7}$$

If the electrons are in a Maxwellian distribution, such that $N_e = N_\infty e^{\chi^*}$, then the nondimensional Poisson's equation can be written

$$\left(\frac{d^2\chi^*}{d\xi^2} + \frac{2}{\xi}\frac{d\chi^*}{d\xi} - e^{\chi^*}\right)(-\chi^*)^{1/2}\xi^2 = -I/I_\lambda \tag{8}$$

where

$$I_\lambda = (2e/m_i)^{1/2}(kT_e/e)^{3/2} \tag{9}$$

This is the equation solved by Allen, Boyd, and Reynolds (ABR). Some results are given in their paper, and more complete results have been computed by Chen.[8] The most extensive investigations of Eq. (8) are those carried out by Laframboise[9] and are plotted in Figs. 3a and 3b.

The importance of the ABR calculation resides not only in the utility of the results for practical applications, but also in the fact that the ABR equation represents the correct limiting equation for $T_i/T_e \to 0$ to the exact equations for a spherical probe, both for the monoenergetic ion approximation treated by Bernstein and Rabinowitz,[7] and the Maxwellian ion distribution considered by Laframboise.[9] The ABR analysis is particularly important for the Laframboise calculations since, because of the nature of Laframboise's numerical scheme, he was unable to carry out calculations in the limit $T_i/T_e = 0$, and thus the ABR

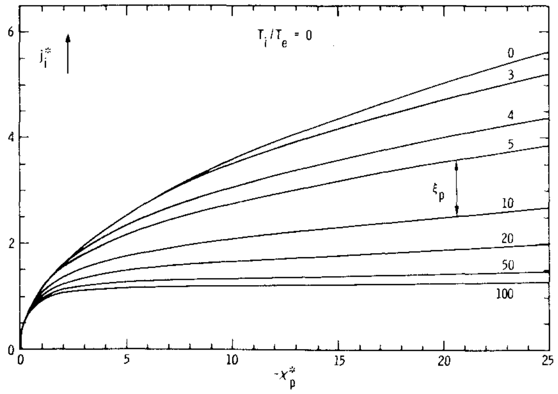

Fig. 3a Ion current j_i* vs probe potential for various ratios of probe radius to electron Debye length; ion-attracting cylindrical probe; $\varepsilon = 0$ (from Laframboise[9]).

values provided the necessary end points for the effect of varying T_i/T_e on the ion current collection by a spherical probe.

An interesting point arises with regard to the cold ion ($T_i/T_e \to 0$) approximation for ion current collection by a cylindrical probe. It was shown by Allen, Boyd, and Reynolds that, in contrast with the spherical probe, it is not correct for the cylindrical probe in the limit $T_i/T_e \to 0$ to assume that the ions start from rest at infinity and move radially inward. The reason for this anomaly has also been discussed by Laframboise[9] and by Lam.[10] Briefly, it has to do with the fact that the limit $E_i \to 0$ is a singular limit in that the angular momentum Ω_i as $r \to \infty$ becomes indeterminate. If Ω_i $(r \to \infty)$ is allowed to have an isotropic distribution, then the Bernstein-Rabinowitz analysis results. If, however, only $\Omega_i = 0$ is permitted, then the radial motion formulation ensues. It is generally accepted that the Bernstein-Rabinowitz formulation which allows for all values of Ω_i is the proper one, and hence the cold-ion model which does not take into account potential barriers will overestimate the current under truly collisionless conditions. However, it also appears that the radial motion approximation for the cylinder may have application to ion current collection under the special situation where ion-ion collisions become important. This point is discussed later on.

Exact Theories for Current Collection by Spherical and Cylindrical Probes in the Collisionless Limit

There are two analyses for current collection by spherical and cylindrical probes under collisionless conditions which can be considered to be exact. One is the analysis of Bernstein and Rabinowitz,[7] who were the first to carry out a complete calculation of probe response including essentially the full range of possible particle orbits and the effect of potential barriers. To make their calculation tractable, they introduced the assumption of monoenergetic ions. Following this, Laframboise[9] and Hall and Fries[11] extended the Bernstein and Rabinowitz formulation to the physically more reasonable case of a Maxwellian distribution of ions in the ambient plasma, and carried out extensive numerical computations of ion and electron current collection for both spherical and cylindrical probes, over a wide range of Debye ratios, $\xi_p = R/\lambda_D$, temperature ratio $\varepsilon = T_i/T_e$, and probe potential χ_p. The calculations of Laframboise and of Hall and Fries are sufficient for establishing the current-voltage characteristics of spherical and cylindrical probes over essentially the entire range of practical conditions of operation, in the collisionless limit. It is not possible in the space allotted here to give an account of the analysis and computational methods employed. We shall give instead only a qualitative description of the method and discuss only the final results obtained by Laframboise, together with several alternative fitting formulas for these results.

It was mentioned earlier in the discussion of the orbital-motion-limited regime that if the Debye ratio $\xi_p = R/\lambda_D$ is not zero, as is required for OML probe behavior, then potential barriers can appear in the effective potential energy which governs the radial motion of the particles, and prohibit certain classes of particles from reaching the probe. There are two main classes of particles which fall in this category. The first consists of particles coming in from infinity, which in the orbital motion limit would be collected by the probe but which encounter an intervening region where the attractive force of the probe is insufficient to overcome the repulsive influence of angular momentum conservation, and thus they fly by the probe and go off again to infinity. In terms of the generalized potential formulation, these particles encounter a potential barrier and are reflected back from it. The second class of particles consists of those which are in planetary-like orbits of bounded variation, that is, orbits (not necessarily closed) which neither intersect the probe nor reach to infinity. These, called trapped orbits, can only be populated by collisions or transient phenomena and there is no a priori way in a steady-state collisionless theory to determine the density of particles in such orbits. However, calculations such as Laframboise's, in which trapped orbits are assumed to be unpopulated, have given results in good agreement with experiment, so apparently the influence of particles in trapped orbits is not important.

Through a detailed analysis of the various classes of particle orbits, Bernstein and Rabinowitz were able to divide $E - \Omega^2$ [cf. Eqs. (3) and (4)] phase space into regions appropriate to the different classes of particles, and to assign distribution functions to each of the regions, under the assumption of monoenergetic, attracted species. The first moment of these distribution functions, which gives the particle densities appropriate to each region, when inserted into Poisson's equation yielded an ordinary differential equation which was solved numerically for selected values of probe current to obtain a family of current-voltage characteristics for both spherical and cylindrical probes. Following the work of Bernstein and Rabinowitz, Laframboise and also Hall developed numerical methods for extending the Bernstein and Rabinowitz method to the case of a Maxwellian distribution for the collected species (either ions or electrons) in the ambient plasma. These results are considered to be the most accurate

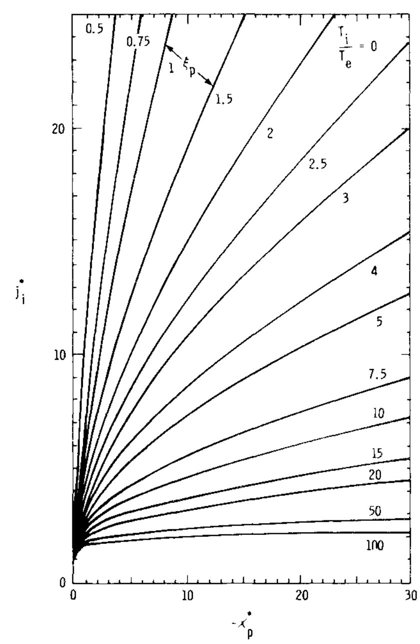

Fig. 3b Ion current j_i* vs probe potential for various ratios of probe radius to electron Debye length; ion-attracting spherical probe; $\varepsilon = 0$; electrons reflected by probe surface; obtained from numerical solution of the Allen, Boyd, and Reynolds equation (from Laframboise[9]).

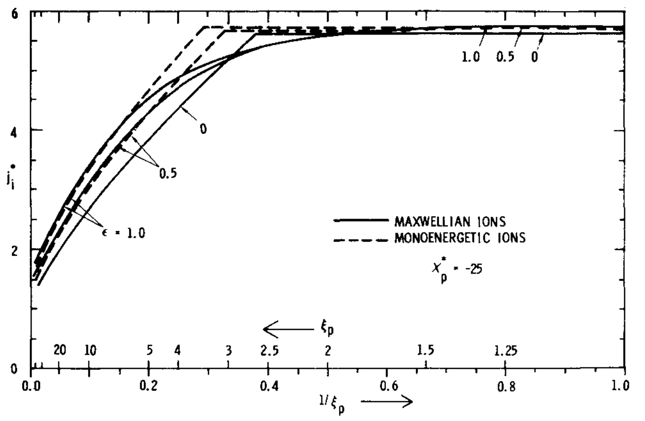

Fig. 4 Ion current to cylindrical probe in a static plasma as a function of ξ_p for $\chi_p^* = -25$, for various values of ε (from Laframboise[9]).

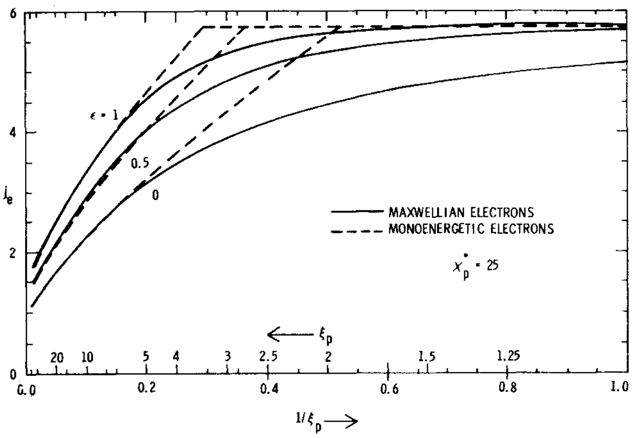

Fig. 5 Electron current to cylindrical probe in a static plasma as a function for ξ_p for $\chi_p^* = 25$, for various values of ε (from Laframboise[9]).

for ion and electron current collection by spherical and cylindrical probes in the collisionless limit. Interestingly, Laframboise found that the Bernstein-Rabinowitz monoenergetic ion model, which he also calculated, gave results generally in very good agreement with those based on the Maxwellian model.

Some typical results obtained by Laframboise for ion and electron collection by cylindrical probes at a probe potential $\chi_p^* = -25$ for several values of ε are shown in Figs. 4 and 5. Several things may be noted in these figures. For ion collection, Fig. 4, the break in the curves for the monoenergetic cases occurs at the onset of the orbital-motion-limit, and for all ξ_p less than the value at the break point, the current is OML, and constant at a value which depends only very slightly on ε, if $|\chi_p^*| > 1$. The current in the Maxwellian case approaches the OML smoothly rather than discontinuously, but in the limit $\xi_p \to 0$, it agrees quite well with the monoenergetic case. When the current is not OML, in the range $\xi_p \gtrsim 5$, the monoenergetic and Maxwellian results also agree quite closely, giving support to the usefulness of Bernstein-Rabinowitz monoenergetic approximation. It is in fact only in the region of transition to the OML that the monoenergetic and Maxwellian results differ appreciably. The results for electron current collection, shown in Fig. 5, exhibit essentially the same features as those for ion collection.

Laframboise's results for the spherical probe are for the most part qualitatively similar to those shown here for the cylindrical probe. One difference between the two cases is that whereas the OML is attained at a finite value of ξ_p for the cylindrical probe, it appears that for the spherical probe the OML is reached only in the limit $\xi_p = 0$, for Maxwellian particles. Also, the monoenergetic and Maxwellian results are not in as good agreement in the limit $\xi_p \to 0$ as they are for the cylindrical case.

Several approximate fits to Laframboise's results have been made. One due to Kiel[12] takes the following form for ion collection by a cylindrical probe, with $Z_i = 1$,

$$j_i^* = F(\varepsilon)\{1 + [f(\varepsilon)/\xi_p^{3/4}](-\chi_p^*)^{1/2}\} \tag{10}$$

where

$$F(\varepsilon) = \varepsilon^{1/2}\{e^{\chi_s}\operatorname{erfc}\chi_s^{1/2} + 2(\chi_s/\pi)^{1/2}\}$$
$$\chi_s = 0.693/\varepsilon \quad \text{for } \varepsilon \leqq 1 \tag{11}$$
$$f(\varepsilon) = (2.18)(1 - 0.2\varepsilon^{0.35})(1+\varepsilon)^{-1/8}$$

A slightly different form of the equation can be found in the paper for the case of electron collection, with $\varepsilon \geqq 1$, and modifications for $Z_i > 1$ are also given. Kiel[13] has also developed a fitting formula for Laframboise's results for the spherical probe.

Another fitting formula for the cylindrical probe has been developed by Peterson and Talbot,[14] which takes the form for both ion and electron current collection

$$j_{i,e}^* = (\beta + |\chi_p^*|)^\alpha \tag{12}$$

with

$$\alpha = a/(\log\xi_p + b) + c\varepsilon^m + d$$
$$\beta = e + \varepsilon\{f + g(\log\xi_p)^3 - l/\xi_p\} + l/\log\xi_p \tag{13}$$

and the constants a, b, c, d, e, f, g, l, and m have the values (assuming that $\varepsilon \leqq 1$) given in the following table.

The Kiel and Peterson-Talbot fitting formulas for the cylindrical probe are about of equal accuracy, agreeing with Laframboise's numerical results generally within less than 3% error. Kiel's formula for the spherical probe is of about the same accuracy, although there is some difficulty in approximating the cold ion limit $\varepsilon \ll 1$, which is the Allen-Boyd-Reynolds case. None of the fitting formulas is accurate in the OML, so they should be used only for $\xi_p \gtrsim 5$. The advantages of these fitting formulas, where they apply, are twofold. The obvious one is that they provide a means for interpolation and extrapolation of the numerical results given by Laframboise. The other is that they provide analytical representations of the ion and electron current which can be used to express the characteristics of double-probe response for finite ξ_p, when the "saturation" currents are voltage-dependent. This extension for the cylindrical probe is discussed in detail by Peterson and Talbot.[14]

A general comment may be made regarding the use of Laframboise's results or the approximations thereto. It will be noted that in order to apply them, one must know both $\varepsilon = T_i/T_e$ and ξ_p. Generally, one can determine T_e from the electron-retarding region of the probe characteristic, if suitable care is taken (cf. Kirchhoff et al.[15]), but T_i must in general be obtained or estimated by some independent means, such as a gasdynamic measurement or a calculation. (An exception to this, related to transient probe response, is discussed later on.) However, in many practical circumstances, $\varepsilon \ll 1$, and the results for the $\varepsilon = 0$ limit can be used with some confidence. Since the

Table 2 Values of constants in Eq. (13)

	a	b	c	d	e	f	g	l	m
Ion collection	2.900	2.300	0.070	−0.340	1.500	0.850	0.135	0.000	0.750
Electron collection	2.900	2.300	0.110	−0.380	−2.800	5.100	0.135	2.800	0.650

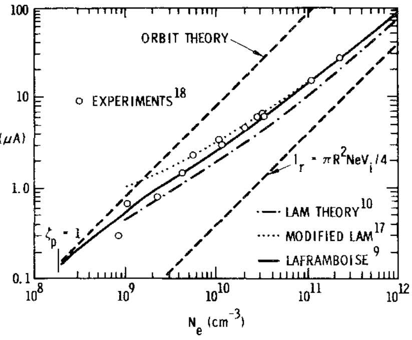

Fig. 6 Measured values of j_i^* for a spherical probe as a function of plasma density as determined by microwaves. Probe radius = 0.025 cm, $\chi_p^* = -20$ (from Chen et al.[18]).

Debye ratio ξ_p involves both T_e and the ambient charged particle density N_∞ which is usually the quantity to be determined, it too is initially unknown, and it would seem that some iteration would be required. However, Sonin[16] has shown how iteration can be avoided if one wishes to determine N_∞ from, say, a single ion current measurement, assuming that the value of ε has been established by some means. Sonin suggests that one choose some fixed potential for the probe which is sufficiently negative that electron current will be negligible, say 10 dimensionless volts below the floating potential, so that $\chi_p^* = \chi_f^* - 10$. (χ_f^* itself is of the order of -5.) He notes that the quantity

$$j_i^* \xi_p^2 = (R^2/\sigma_o)(2m_i/e)^{1/2}(e/kT_e)^{3/2}(I_i/A_p) \tag{14}$$

is a function only of T_e and the measured probe current density (I_i/A_p), even though j_i^* itself is a function of ξ_p, ε, and χ_p^*. Sonin, therefore, suggests that for a chosen fixed probe potential, say $\chi_p^* = \chi_f^* - 10$, one constructs curves of j_i^* vs $j_i^* \xi_p^2$ for various values of ε. Since $j_i^* \xi_p^2$ is known from the measurement, j_i^* can be read from such a plot, and thus ξ_p^2 can be determined. Of course, once ξ_p is known, so is N_∞. This procedure has been used with satisfaction by many workers. The procedure implies that T_e has been determined by some means, presumably from the slope of the retarding field portion of the probe characteristic. Hence, even if only one point on the ion current curve is used, the full characteristic will usually be required. Obtaining the full probe characteristic whenever possible is good practice in any case, since otherwise possible anomalies in probe response such as are discussed later may go undetected.

We have thus far made no mention of the several analytical studies that have been carried out for cylindrical and spherical probes, in particular the asymptotic analysis of Lam.[10,17] Lam's analysis, which was carried out before Laframboise's numerical results became available, is based on the Bernstein-Rabinowitz monoenergetic ion model, and involves the use of matched asymptotic expansions in the limit $\xi_p \gg 1$, for the quasi-neutral, intermediate and sheath regions surrounding spherical and cylindrical probes. This analysis provides much insight regarding the detailed structure of the several regions mentioned, but for the practical purpose of constructing a current-voltage characteristic it is now better to use the Laframboise results than Lam's earlier asymptotic analysis. An exception to this might be found if it were desired to examine the detailed nature of the approach to the $\xi_p = \infty$ limit, since this is the region where Lam's analysis is most accurate, and also where (for $\xi_p > 100$) essentially no numerical results exist. Alternatively, one may wish to use the results of the Lam analysis for values of $|\chi_p^*|$ in excess of 25, the limit of the Laframboise calculations, in preference to the fitting formulas.

Before leaving the subject of probe response in the collisionless regime, it is appropriate to say a few words about experimental verification of the theory. There have been several attempts to verify the Laframboise results, in both quiescent and flowing plasmas. In the case of a quiescent plasma, one of the most satisfactory verifications of the theory is that which was carried out by Chen, Etievant, and Mosher.[18] They tested both cylindrical and spherical probes in a potassium plasma and compared the inferred charged-particle concentrations with those determined simultaneously by microwave measurements. Because of the nature of the experiment, the cylindrical probes were operated mainly in the orbital-motion regime, but the spherical probes operated in a regime which spanned a considerable range of ξ_p, from a value low enough for the current to approach the orbital limit at $\xi_p = 0$ (for Maxwellian ions) to a value of $\xi_p \sim 30$, where considerable departure from the OML current is found. For both types of probes good agreement with the Laframboise results were found over the entire range of operation. The results of Chen et al.[18] for spherical probes are shown in Fig. 6. As can be seen from this figure, the Lam analysis was found also to give accurate predictions in the range of its validity, which is $\chi_p \xi_p^{-4/3} = 0(1)$, with $\xi_p^2 \gg 1$.

Attempts have also been made to verify Laframboise's results for cylindrical probes in flowing plasmas, under the assumption that a cylindrical probe aligned with the flow direction will, if all relevant mean free paths are significantly greater than the probe radius, respond in the same fashion as a cylindrical probe in a stationary plasma. Under certain conditions this assumption has proved to be correct, and results in agreement with the Laframboise calculations have been found as for example by Sonin,[16] Dunn and Lordi,[19] and by Lederman et al.[20]

In the case of Sonin's measurements, as will be discussed in more detail later, apparent agreement with Laframboise's calculations was found except in the orbital-motion regime, but it was subsequently determined that ion-ion collisions played an important role in the measurements. Dunn and Lordi found good agreement between their data and the Laframboise results in a small range of the orbital-motion regime where $\xi_p \gtrsim 2$, but found that the measured currents exceeded the theoretical values for lower values of ξ_p. Somewhat similar results were obtained by Lederman et al. However, both the Dunn-Lordi and the Lederman et al. results were subsequently determined to contain an end-effect contribution, about which more will be said later. The resolution of these ion-ion and end-effect contributions has eventually led to the substantiation of the use of the Laframboise results for aligned cylindrical probes in flowing collisionless plasmas, as well as in stationary plasmas. Further discussions of the end-effect will be given in the section "The Effect of Flow."

The Laframboise results for electron current collection by cylinders has been substantiated also by experiment. Dunn[21] found excellent agreement between number densities obtained by ion-current-collecting and electron-current-collecting cylindrical probes, for $\xi_p > 1$, in both shock tunnel freestream and flat-plate boundary-layer flows.

Collisional Effects on Probe Response

When the ion mean free paths for collisions with either neutral gas atoms or with other ions are not large compared to the probe radius, then the collisionless theory discussed in the previous section no longer applies. Several attempts have been made to analyze in a rigorous self-consistent fashion spherical probe response under the influence of ion-neutral collisions, in a stationary plasma. One of the earliest of these is the work of Wasserstrom, Su, and Probstein,[22] and this was later followed by the studies of Self and Shih,[23] Chou, Talbot, and Willis,[24] and by Bienkowski and Chang.[25] From an analytical point of view, the kinetic theory approach of Chou, Talbot, and Willis (CTW) is perhaps the most rigorous, although the numerical work necessary to generate a useful family of probe characteristics is prohibitive.

The CTW formulation is based on a moment method solution of the Boltzmann equation, using a Krook-type model for the collision integral. Essentially, all possible ion trajectories are considered and the regions of energy-angular momentum space are divided in the same fashion as was done by Bernstein and Rabinowitz. The moment method is essentially the Lees "Two-stream Maxwellian" one. The idea is that a distribution function is assumed which satisfies the collisionless limit exactly, and collisional effects are accounted for in some average sense by taking sufficient moments of the Boltzmann equation to determine the various unknown parameters in the assumed distribution function.

The cases calculated by CTW agree with Laframboise's results in the collisionless limit. It was found that as the ion-neutral Knudsen number λ_{in}/R is decreased, the ion current decreased relative to its collisionless value at the same conditions of ξ_p and χ_p, and the sheath region was found to extend further into the plasma. Enough cases were calculated to establish these trends, but not enough to cover a sufficient range of the parameters important to our experimentalist.

The theory of Bienkowski and Chang[25] for a spherical probe, like the CTW theory, takes into account essentially all possible charged particle orbits within the framework of a moment method. However, "exact" results were obtained only for the asymptotic thin sheath limit $\xi_p^{-4/3}\chi_p \ll 1$, $\chi_p \gg 1$. Although suggestions are given by Bienkowski and Chang for approximate procedures which could be used to relax these conditions, the range of conditions covered by the analysis is still somewhat limited.

Somewhat simpler approaches were used by Wasserstrom et al.[22] and by Self and Shih.[23] Wasserstrom et al. used an approach similar to CTW, but considered only straight-line trajectories. Their results and the CTW results agree in the limit of very small $|\chi_p|$, but the Wasserstrom et al. results become progressively less accurate as the probe potential is made more negative. Moreover, their analysis is most applicable to the near-continuum limit, whereas the only results obtained by CTW were in the near-collisionless regime, so a comparison between the two approaches is not too meaningful. The approach of Self and Shih is essentially a modification of the Allen-Boyd-Reynolds formulation for cold ions (radial motion), to account for collisions. The modification takes the form of a collisional friction term, $v_i v_r$, in the ion-momentum equations with $v_s(-\chi^*)^{1/2}$ restored to v_r, which is solved together with the ion continuity equation $I_i = 4\pi r^2 N_i v_r$, and Poisson's equation, Eq. (8). Some experiments, unfortunately limited by the nature of the apparatus to a rather narrow range of v_i/ω_{pi}, are also reported which appear to confirm the predictions of the Self-Shih theory. In a subsequent paper Shih and Levi[26] present an approximate form of the Self and Shih results, and extend the analysis to the case of a cylindrical probe. The approximation used restricts the applicability of the results to small collisional effects.

Although the calculations required in the Chou-Talbot-Willis analysis makes it difficult to use, several simple approximate results have been developed. Talbot and Chou[27] started with the general expression obtained by CTW for the spherical probe

$$\frac{j_{i,\infty}}{j_i} = 1 + \frac{R}{\lambda_i} j_{i,\infty} \int_o^1 e^{\chi} d\left(\frac{R}{r}\right) \tag{15a}$$

$$= 1 + (R/\lambda_i) j_{i,\infty} I_c \tag{15b}$$

where j_i is the normalized ion-current density, and $j_{i,\infty}$ is its value in the collisionless limit. They then constructed approximations for the integral I_c in Eq. (15b), in the limits of collisionless and collision-dominated flows which we shall denote, respectively, by $I_{c,\infty}$ and $I_{c,o}$. Since $I_{c,\infty}$ and $I_{c,o}$ are of the same order of magnitude, they used a simple Knudsen number interpolation formula of the form

$$I_c = I_{c,\infty} + (I_{c,o} - I_{c,\infty})/[1 + (\lambda_i/R)] \tag{16}$$

to express I_c in the range between the two limits. Since $j_{i,\infty}$ is known from Laframboise's results as a function of χ_p, R/λ_D, and ε, and I_c has been constructed as a function of these parameters, if the Knudsen number λ_i/R is known then the probe current j_i in the presence of collisions can be obtained. This approach was followed also for ion-current collection by a cylindrical probe by Talbot and Chou.

An even simpler approach was used by Schulz and Brown,[28] Sutton,[29] and later by Thornton[30] who gave some additional justification for the procedure. In effect, they used a simple interpolation formula for the ion current itself (Sutton used this for the electron current) of the form

$$j_i = \frac{j_{i,\infty}}{1 + (j_{i,\infty}/j_{i,o})} \tag{17}$$

Since $j_{i,o}$, the collision-dominated or diffusion-controlled limit of ion-current collection, is linear in λ_i, when $\lambda_i \to \infty$ then $j_i \to j_{i,\infty}$, and when $\lambda_i \to 0$ then $j_i \to j_{i,o}$, and one can see that the approach is similar to the Knudsen number interpolation used by Talbot and Chou. From Eq. (17) it follows that the current collection to a probe of any geometry in the transition regime can be estimated if the collisionless and collision-dominated currents for that geometry are available. In this connection it is worth noting that better results for the collision-dominated limit for the cylinder are now available from the work of Inutake and Kuriki,[31] who applied the method of Su and Lam[32] to the case of an ellipsoidal probe at highly negative potential.

The usefulness of these approximate transition regime theories has been tested experimentally by Kirchhoff, Peterson, and Talbot,[15] Dunn and Lordi,[19] and Thorton.[30] Thornton found that the Talbot-Chou, Self-Shih,[23] and his own method were all in quite good agreement with one another and represented well his data for both cylindrical and spherical probes. Probably the Thornton method is the easiest to use. In the application of his method to a probe measurement in which the ion-number density is initially unknown, it is of interest to point out that, just as in the case of the collisionless limit, the quantity $j_{i,o}\xi_p^2$ (or $j_{i,o}{}^*\xi_p{}^2$) is independent of N_i. Hence one can construct transition regime curves of $j_i\xi_p^2$ vs ξ_p for selected values of Knudsen number, probe potential, and ion-electron temperature rates, and follow the same approach described by Sonin to obtain N_i from a probe measurement in the transition regime without iteration.

The effect of electron-neutral collisions in electron saturation current to cylindrical and spherical probes in a stationary plasma has been examined by Peterson,[33] using the Talbot-Chou approach and obtaining similar results. However, in the electron-retarding region of the probe characteristic, which is generally used to infer the electron temperature, the effect of electron-neutral collisions on the probe current is less well understood. There is evidence, both theoretical and experimental, that the classical $j_e = \exp(-e\phi_p/kT_e)$ behavior is sufficiently altered by collisions so that the method of obtaining T_e from the slope of a plot of $\log j_e$ vs probe potential ϕ_p no longer holds. Kirchoff et al.[15] have discussed the effect of electron-neutral collisions on the determination of electron temperature, and have concluded, on the basis of both theory and experiment, that a double cylindrical probe is less sensitive than a single one to collisional effects, and hence that double probes may often be used to determine electron temperature under conditions where single probes may give spurious results.

All of the aforementioned analyses for the effect of collisions on current collection by probes are concerned with charged-neutral collisions, which act to reduce the current below that of the collisionless limit (say, the Laframboise value). The effect of ion-ion collisions, under circumstances where ion-neutral collisions are negligible, appears to be just the reverse, at least for the case of the cylindrical probe. This is the conclusion reached as a result of a reappraisal of the measurements made by Sonin[16] on cylindrical probes aligned with the flow direction under conditions where $\lambda_{in}/R \gg 1$ but $\lambda_{ii}/R < 1$. These data are reproduced in Fig. 7. Although a satisfactory independent measurement of the freestream ion density was not available to Sonin, he observed that his data followed the Laframboise prediction for the relative variation of $j_i{}^*\xi_p{}^2$ with $j_i{}^*$, except when the orbital motion limit was reached. The Laframboise calculation presented in this fashion predicts that when $j_i{}^*\xi_p{}^2$ is

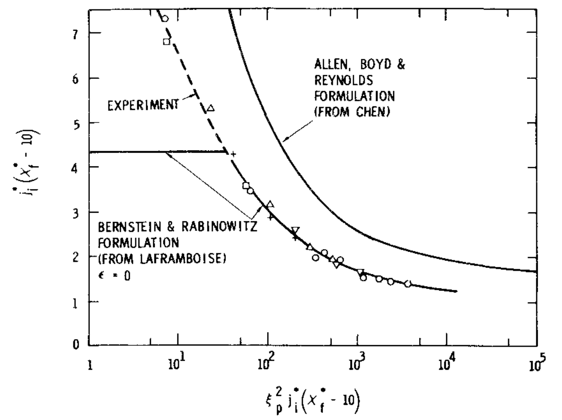

Fig. 7 Sonin's data for ion current collection by an aligned cylindrical probe in a flowing plasma (from Sonin[16]).

reduced to the value corresponding to the onset of orbital motion, the current j_i^* reaches a constant value and does not further increase with additional reduction in $j_i^*\xi_p^2$. It can be seen from Fig. 5 that Sonin found that j_i^* continued to increase as $j_i^*\xi_p^2$ decreased, and he attributed this to ion-ion collisional effects, since he noted that λ_{ii}/R was not large in his experiments. This conjecture appears to be correct. However, a reassessment of these data by Hester and Sonin[34] suggests that ion-ion collisional effects may be important not only in the orbital-motion-regime, but over the entire range of $j_i^*\xi_p^2$, and that the data presented in Fig. 7 should be shifted upward to lie on the prediction for cylindrical probe current based on the Allen-Boyd-Reynolds model[1,8,9] of radial motion.

The Effect of Flow on Aligned Cylindrical Probes under Collisionless Conditions

As was remarked earlier, it seems plausible to assume that a cylindrical probe aligned with the flow direction will exhibit the same characteristics as predicted by Laframboise for cylindrical probe in a stationary collisionless plasma, if all the relevant Knudsen numbers $\lambda_{\alpha\beta}/R$ are $\gg 1$, or in the case where only λ_{ii}/R does not satisfy this criterion, the characteristics predicted by the Allen-Boyd-Reynolds model. This plausibility argument would seem to be substantiated by the experiments of Dunn and Lordi[19] and by Graf and De Leeuw.[35] However, there is a situation under which the argument does not apply. This is the situation when an "end-effect" becomes important, and an additional parameter enters, the ratio l/λ_D of the probe length to the Debye length. One may also consider this parameter as arising from the introduction of the aspect or "fineness" ratio l/R, in the form of the product $(l/R)(R/\lambda_D)$.

The end-effect was first discussed by Bettinger and Chen[36] in connection with measurements made aboard Explorer 17. A sharp peak in the ion current was observed when the probe was precisely aligned with the flow. The same phenomenon was observed even more clearly in laboratory experiments carried out by Hester and Sonin.[37]

Several analyses of this effect have been carried out. First, Bettinger and Chen carried out an approximate analysis for the response of a cylindrical probe at angle of attack. Then Hester and Sonin produced a numerical solution for the aligned cylindrical probe. Sanmartin[38] has produced an analytical study which parallels Hester and Sonin's numerical study, and also has been able to carry out a more accurate analytical study of the angle-of-attack problem treated in approximate fashion by Bettinger and Chen.[39]

Before discussing the results of these studies, in particular the results of Hester and Sonin, we will briefly describe in physical terms the nature of the end-effect. It occurs for $\xi_p \ll 1$, that is, when the sheath radius becomes significantly greater than the probe radius and under conditions when, in a quiescent plasma, the probe would be operating in the OML regime. Now, Langmuir and Mott-Smith derived the approximate expression for current collected by a cylindrical probe at angle of attack θ to a flow with velocity U

$$I_{i,\infty} = 2eN_\infty URl\left[\sin^2\theta - \frac{e\phi_p}{m_i U^2/2}\right]^{1/2} \tag{18}$$

which is identical to the OML Eq. (6a) in the limit $|e\phi| \gg kT_i$. Equation (18) gives the current one would expect for an aligned cylindrical probe wih $\xi_p \ll 1$, and does not account for the observed peak in the current. However, Eq. (18) is based on the model of current collection governed by an impact parameter, b, as shown schematically for a probe in a transverse orientation in Fig. 8. Now, a probe in the aligned orientation, as shown also in Fig. 8, can collect particles not only through the lateral sheath surface area, but also through the end of the sheath. If the sheath radius which is proportional to λ_D is large enough, and if $U \gg (kT_e/m_i)^{1/2}$, then a significant number of ions can reach the probe with velocity U through the end of the sheath as well as by transverse motion across the cylindrical boundary of the sheath with a velocity, according to the Bohm condition, of the order of $(kT_e/m_i)^{1/2}$, and this in fact is the phenomenon responsible for the observed peak in ion current. Hence, the end-effect should depend on the parameter

$$\tau_l = \frac{l}{\lambda_D}\frac{(kT_e/m_i)^{1/2}}{U} \tag{19}$$

since the relative importance of the current contribution from the end of the sheath should depend on the ratio of the product of the transverse velocity of the particles and the lateral surface of the sheath to the product of directed flow velocity and the end area of the sheath. One may anticipate that for $\tau_l \gg 1$, the end effect will be negligible. Now, if the probe is not oriented exactly at $\theta = 0$, but turned at a slight angle to the flow direction, even though τ_l is not large, many of the oncoming ions which enter the end of the sheath will have sufficient angular momentum to escape and will not be collected. This is the reason why the end effect, when it appears, is associated with a very small angular range around $\theta = 0$, and there is a rapid drop in current to the infinite probe value given by Eq. (18) for small θ. We can expect that the effect will also be a function of $\varepsilon = T_i/T_e$, since the ion temperature is a measure of the random ion velocities in the lateral direction. It is in fact this end effect, as was shown by Hester and Sonin, which is responsible for the apparent discrepancies between the Dunn and Lordi and Lederman et al. data and the Laframboise theory in the orbital motion limit. When account is taken of the effects, then these data are found to agree with the theory throughout the entire range of ξ_p covered by the data.

Hester and Sonin approach the analysis of the end effect for aligned probes ($\theta = 0$) by formulating the problem as a one-dimensional unsteady one, in which the normalized time variable $\tau = (x/l)\tau_l$ is related to the flow speed and distance along the probe in the steady flow problem. At time $\tau \leq 0$ the space around the probe is taken to be divided into annular regions, each populated with a discretized approximation to a two-dimensional Maxwell-Boltzmann distribution. The (negative) probe potential is assumed to be applied to the probe at $\tau = 0$, and the subsequent

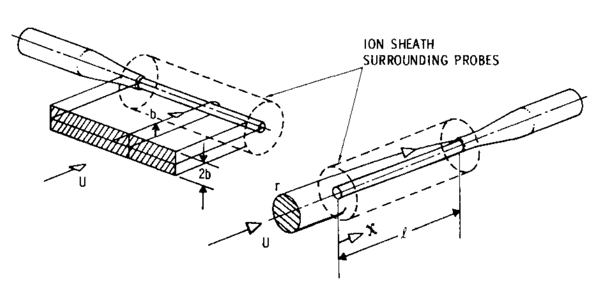

Fig. 8 Schematic of sheath around probes, illustrating the origin of the end-effect (from Hester and Sonin[34]).

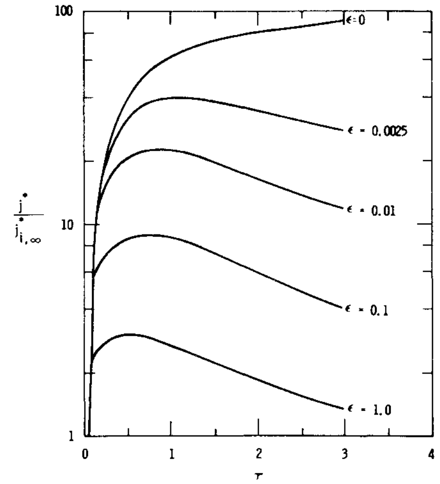

Fig. 9 Ion-current density to aligned cylindrical probe as a function of dimensionless time τ, for $\xi_p = 10^{-2}$ and $\chi_p{}^* = -15$ (from Hester and Sonin[34]).

motions of groups of particles having selected values of angular momenta are calculated by an iterative solution of the particle equation of motion and Poisson's equation. The current j_i to the probe as a function of time is calculated directly from the rate of accretion of charge due to particles striking the probe.

Typical results of the Hester-Sonin calculation are shown in Fig. 9. Figure 9 shows the instantaneous current density to a probe as a function of τ, for $\xi_p = 10^{-2}$, and for $\chi_p{}^* = -15$. The current starts at zero at $\tau = 0$, because of the way the problem was formulated, and rises rapidly to a maximum, around $\tau = 1$, for all finite values of $\varepsilon = T_i/T_e$. Although the calculations were not carried beyond $\tau = 3$, the currents for all $\varepsilon > 0$ decrease from their maxima as τ increases, and for sufficiently large τ, $j_i/j_{i,\infty}$ would approach unity, where $j_{i,\infty}$ represents the current in the limit $\tau \to \infty$. The current density in the limit $\tau \to \infty$ corresponds in the steady flow problem to the current density to the probe as $x \to \infty$, and is therefore equivalent to the average current density collected by an infinitely long probe where $\tau_l \to \infty$.

An intriguing inference can be drawn from the transient response shown in Fig. 9. If one has a probe situation which under steady-state conditions corresponds to $\tau_l \gg 1$ (negligible end effect), then one can determine the ion density according to Eq. (19), provided, of course, $\xi_p \lesssim 1$. However, if this same probe is operated in a transient fashion, that is, the probe potential $\chi_p{}^*$ is suddenly applied at $\tau = 0$ and maintained at that value, the current overshoot which accrues during the establishment of the steady state will be a very strong function of ε, and thus the magnitude of this overshoot might be used to determine the ion temperature. The numerical results for this very interesting transient probe response have now been supplemented by an analytical treatment due to Sanmartin,[38] to which the reader is referred for further details.

The end effect we have been discussing has to do with collisionless plasma flow past an aligned cylindrical probe. However, it appears that collisional effects may also produce phenomena similar to those observed under collisionless conditions for small values of τ_l, in particular a peak in ion current in the neighborhood of zero angle of incidence, rather than a monotonic variation with incidence of ion current such as is predicted by Eq. (18). This was, in fact, first noted by Sonin,[16] who found that, unlike the collisionless end effect, the peak was independent of probe aspect ratio l/R. Hester and Sonin[34] later gave an explanation for this peak as being due to ion-ion collisions; in effect, the probe when aligned with the flow will, because of ion-ion collisions, collect ion current more or less according to the Allen-Boyd-Reynolds radial flow model, but when the probe is set at any significant incidence to the flow, the momentum of the flow will dominate over ion-ion collision effects and the current collected by the probe will be governed by orbital motion considerations. Recently, Jakubowski[40] has reported ion-current measurements with cylindrical probes aligned with and at incidence to the flow, similar to the experiments of Sonin. Like Sonin, he also observed an ion-current peak in the neighborhood of zero angle of incidence which was independent of aspect ratio. Although Jakubowski suggests that the current peak may be due to "pre-sheath" ion-neutral collisions rather than to ion-ion collisions as advanced by Hester and Sonin, it is probable that the latter interpretation is the correct one. It is perhaps worth mentioning that when λ_{in}/R decreases to values of the order of unity, or less, collisional effects on aligned cylindrical probes in a flowing plasma become much more complicated than in the case of a stationary plasma, as was found by Kirchhoff et al. since both the neutral gas flowfield and the ion motions are modified by collisions, and thus Mach and Reynolds number effects must be taken into account also.

In some applications involving the use of probes in high-speed flowing plasmas under collisionless conditions at small values of ξ_p, it may be preferable to use cylindrical probes oriented transverse to the flow direction, particularly in the case where $kT_e/m_i \gg U^2 \gg kT_i/m_i$. For ion collection by such probes, the simplest result, valid in the infinite sheath limit $\xi_p \to 0$, is the one by Langmuir and Mott-Smith, Eq. (18), already cited. Various attempts have been made to obtain modifications to this result which would account for the effects of finite sheath thickness, or, stated alternatively, reduction in the value of the impact parameter below its orbital value due to finite penetration of the probe's electric field. Among these attempts are the analyses of Clayden,[41] Smetana,[42] Kanal,[43] and Tan.[44] All of these analyses contain certain ad hoc assumptions which are introduced to deal with the very difficult problem posed by the nonsymmetric character of the potential distribution around the probe and the unknown (and also nonuniform) sheath thickness. Probably Smetana's results are as useful as any if the flow speed is not too high, and they have been used with reasonable success by several investigators.[45]

There are, however, several ways in which the behavior of a transverse cylindrical probe in a high-speed plasma flow differs from an aligned cylindrical probe, even neglecting such complications as end effects or collisional phenomena. In the limit $m_i U^2/kT_e \gg 1$, the ion current, according to Eq. (18), will be given simply (with $\theta = \pi/2$) by

$$I_i = 2eN_\infty URl \tag{20}$$

which is in effect what Clayden showed. However, the electron temperature cannot, in general, be determined directly from a plot of $\log j_e$ vs probe potential (a higher value than the true electron temperature is obtained which must be corrected for velocity effects) nor can the plasma potential be readily identified from a change in slope of the probe characteristic. And finally, and perhaps most interestingly, electron-current saturation fails to occur under conditions where it would be present for a probe in a stationary plasma, because in essence the shielding effect of the probe electron sheath is destroyed by the high-speed ion flow. These phenomena have been demonstrated experimentally by Koopman,[46] Segall and Koopman,[47] and Fournier.[48]

One of the more important applications of probes in flowing plasmas has to do with their use on sounding rockets and satellites for the measurement of charged particle densities at high altitudes. Many probe configurations have been employed, and several theories have been developed which are applicable to the particular conditions associated with such measurements. Unfortunately, space does not permit us to discuss these in any detail, and we can only refer the interested reader to Refs. 49–52, as being typical of the work in this area.

Summary

The following are some of the main conclusions which can be drawn concerning the response of cylindrical and spherical electrostatic probes in the collisionless and transitional regimes.

1) For cylindrical and spherical probes operated under completely collisionless conditions in a stationary plasma, the Laframboise[9] theory gives an accurate description of the probe response, both for ion and electron currents and can be used to obtain the electron density and temperature from the probe current-voltage characteristic curve. The monoenergetic ion model used by Bernstein and Rabinowitz[7] is a very good approximation to the Maxwellian model used by Laframboise.

2) The cold-ion (or radial motion) model of collisionless ion collection introduced by Allen, Body, and Reynolds[6] is a valid limiting condition for a spherical probe in stationary plasma, when $T_i/T_e \to 0$. The radial motion model is not a correct limiting condition for a cylindrical probe in a stationary plasma, when $T_i/T_e \to 0$, and the Bernstein-Rabinowitz results must be used instead. However, the radial motion model appears to be a good representation for cylindrical probe ion-current collection when ion-ion collisions cannot be neglected.

3) The effect of ion-neutral collisions on ion-current collection by spherical and cylindrical probes in the transitional regime in a stationary plasma can be estimated with satisfactory accuracy by means of a simple interpolation formula if the collisionless and collision-dominated limits of the probe ion current are known, and this interpolation formula can thus be used to determine charged particle density from the probe ion current in the transitional regime. No such simple method exists for obtaining electron temperature from the electron-retarding portion of the characteristic curve of a single probe in the transitional regime; however, double probes appear to be less susceptible than single probes to collisional effects in the transitional regime and may often be used for electron temperature determination in this regime when single probes give spurious results.

4) For flowing plasmas under collisionless conditions, cylindrical probes aligned with the flow direction give the same current-voltage characteristics as cylindrical probes in stationary plasmas in the collisionless regime, provided the end-effect parameter $\tau_l \gg 1$. For small values of τ_l, the ion current to an aligned cylindrical probe in a flowing plasma will exceed the orbital-motion-limited value because of additional ion flux reaching the probe through the end of the sheath. This end effect is limited to a quite small range of angle of incidence centered around the aligned position. Under conditions where $\tau_l \gg 1$ and the end effect is negligible, ion-ion collisions can also produce a similar peak in ion-current collection by a cylindrical probe in a small range of incidence centered about the aligned position. This effect is independent of τ_l, and can thus be differentiated from the end effect by varying the probe fineness ratio l/R, which is proportional to τ_l.

5) Transitional effects on probes in flowing plasmas are more complex than for probes in stationary plasmas, and adequate methods for interpreting probe characteristics under flow conditions in the transitional regime are not available. A simple interpolation scheme such as has been found to be useful for probes in stationary plasmas fails under flow conditions for ion-neutral Knudsen numbers less than about unity.

6) Cylindrical probes oriented transverse to the flow direction and operated in the collisionless regime can be used to good effect in very high-speed flows. The interpretation of the ion-current collected by such probes is particularly simple. Electron-current collection, however, differs markedly from that found in stationary plasmas, in that the shielding effect of the electron sheath is obliterated by the high-speed ion flow, with the result that electron-current saturation does not occur. Electron temperatures can be obtained from the electron-retarding portion of the probe characteristic provided a velocity correction is applied.

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Part 2. Continuum Probes

Introduction

THE effects of collisions in the sheath formed around an electrostatic probe were discussed in some detail in part 1 of this survey paper. In part 2, we will review the status of continuum electrostatic probes which represent the limit of "many collisions" within the sheath. As outlined in the Introduction of part 1, one can identify two regimes of continuum probe operation and one hybrid case depending on the relative magnitude of smallest mean-free-path λ and the Debye length λ_D. These are: $L \gg \lambda_D \gg \lambda$—collisional thin sheath; $\lambda_D \geqq L \gg \lambda$—collisional thick sheath; and $L \gg \lambda \geqq \lambda_D$—collisionless thin sheath (dense case). Here L is a characteristic length particular to the problem at hand. The last regime is the hybrid case (sometimes alluded to as the dense case) where the sheath can be described by the collisionless considerations discussed in part 1, but the motion of carriers in the bulk of the plasma is determined from the continuum flow equations. The first two are collision-dominated throughout the plasma and the motion of the carriers is determined by processes of convection, diffusion, and mobility, governed by the continuum equations, Eqs. (1–4).

In part 2 we will discuss the first two regimes in detail. The third will be mentioned briefly. We will develop expressions for probe current collection where possible, display numerical solutions for probe characteristics and make comparison with experimental data where such data exist.

We will limit our discussion to weakly ionized plasmas which will enable us to decouple the fluid mechanical from the electrical characteristics of the flow. The gas velocity, density, and temperature fields are, therefore, presumed to be known and the quantities to be determined are charged particle densities, the electric field in the plasma, and the electron temperature. A brief mention will be made of special probe topics which deal with surface phenomena and turbulent plasmas.

Part 2 Some Physical Considerations

The relative importance of convection, diffusion, mobility, and charge generation on current collection are determined by nondimensional parameters that arise naturally, by nondimensionalizing Eqs. (1–4) as will be shown later on. To put the